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Quantum Measurement, Gravitation, and Locality *

D. V. Ahluwalia

MP-9, MS H-846, Nuclear and Particle Physics Research Group

Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA†

Abstract

It is argued that when measurement processes involve energies of the order of the Planck scale, the fundamental assumption of locality may no longer be a good approximation. Idealized position measurements of two distinguishable spin-0 particles are considered. The measurements alter the space-time metric in a fundamental manner governed by the commutation relations $[x_i\, p_j] = i\hbar\,\delta_{ij}$ and the classical field equations of gravitation. This *in-principle* unavoidable change in the space-time metric destroys the commutativity (and hence locality) of position measurement operators.

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†E-mail address: AV@LAMPF.LANL.GOV

The purpose of this brief essay is to make an *in-principle* remark on the fundamental assumption of *locality* in quantum field theories [1] and its interplay with the measurement process and gravitation. The essential philosophy of this essay is to enhance the quantum mechanical and gravitational effects and ignore the lowest-order *classical* effects (i.e., those effects that do not depend on $\hbar$). We will see that the assumption of locality is deeply connected with gravitation and the measurement process. If *all* gravitational effects (irrespective of whether classical and quantum-mechanically-induced) are ignored, locality is recovered. The remarks that we present are seemingly trivial, but in view of their possible relevance, we take the liberty of presenting them in this brief essay.

To give a precise definition to locality, let us note with Schwinger [2] that:

> "A localizable field is a dynamical system characterized by one or more operator functions of space-time coordinates, $\Phi^\alpha(x)$. Contained in this statement are the assumptions that the operators x_μ, representing position measurements, are commutative,
>
> $$[\,x_\mu\,,\,x_\nu\,] \,=\, 0 \quad , \tag{1}$$
>
> and furthermore, that they commute with the field operators,
>
> $$[\,x_\mu\,,\,\Phi^\alpha\,] \,=\, 0 \quad , \tag{2}$$
>
> so that
>
> $$\langle x|\Phi^\alpha|x'\rangle \,=\, \delta(x-x')\,\Phi^\alpha(x) \quad . \tag{3}$$
>
> The difficulties associated with current field theories may be attributable to the implicit hypothesis of localizability."

In reference to commutativity of the position measurements, expressed by Eq. (1), underlying the "hypothesis of localizability," we consider two neutral spin-0 particles of masses m_1 and $m_2\,(> m_1)$. For the purposes of the following discussion, it would be useful to keep the following idealized picture of the world in view. The world consists of two particles. All measuring devices have no other effect except to introduce the quantum-mechanically-required perturbations consistent with the fundamental commutation relations: $[x_i\ p_j] = i\hbar\,\delta_{ij}$. We now claim that we know, as a result of some appropriate measurement, $\mathcal{M}_1$, that particle-1 is confined to a sphere of radius $R_1 \ll \hbar/(m_1 c)$ centered at $\vec{x}_1$; while the space-time coordinates of particle-2 are completely unknown. Now a time $\Delta t \ll \hbar/(m_1\, c^2)$ later, we make a second measurement, $\mathcal{M}_2$, such that particle-2 is confined to a sphere of radius $R_2 \ll \hbar/(m_2\, c)$ centered at $\vec{x}_2$. The measurement $\mathcal{M}_2$, via the fundamental uncertainty relations $[x_i, p_i] \sim i\hbar\,\delta_{ij}$, imparts certain momentum to particle-2 resulting in a local energy density

$$\begin{aligned}\rho_2(r_2) &\gtrsim \frac{3\,\theta(r_2\ -\ R_2)}{4\pi R_2^3}\left[m_2^2c^4\ +\ \beta\frac{\hbar^2c^2}{R_2^2}\right]^{1/2} \quad , \\ &\gtrsim \frac{3\,\theta(r_2\ -\ R_2)}{4\pi R_2^3}\,\frac{\sqrt{\beta}\hbar c}{R_2}\ , \quad \text{for } R_2 \ll \hbar/(m_2 c) \quad , \end{aligned} \tag{4}$$

where r_2 equals the radial coordinate distance with $\vec{x}_2$ as origin, β is a geometrical factor of the order of unity, and $\theta(r)$ is the usual step function. We shall assume that the two particles have separations (of course, only *after* the measurements are made!) $|\vec{x}_1 - \vec{x}_2| \gtrsim \hbar/(m_1 c)$.

The assumptions $R_{1,2} \ll \hbar/(m_{1,2}\, c)$, etc., are made to keep possible quantum mechanical overlap of wave functions of particle-1 and -2 to a minimum and to enhance purely *quantum* mechanical effects arising solely from the measurement process. The assumption $m_2 \neq m_1$ avoids complications that may arise from indistinguishability of the particles. The particles are assumed to have spin-0 to avoid (gravitational) Thirring-Lense [3] interaction. In order to keep our arguments as simple as possible, we refrain from incorporating uncertainties that arise from the specification of the time variable. The essential character of conclusion that follows is, however, expected to remain unaltered if all, or some of, these assumptions are relaxed.

Define $\rho_1(r_1)$ in a similar fashion to $\rho_2(r_2)$ above. Consider the setup such that in the region $r_1 \leq R_1$ and $r_2 \leq R_2$ we have $\rho_2(r_2) \gg \rho_1(r_1)$. Then, as a result of inherently quantum mechanical perturbation in momentum of a particle by confining it to a finite region of space, we are *forced* to induce a local modification of space-time structure. Explicitly, we see this via the classical field equations of Einstein and Eq. (4). In the spirit of the philosophy outlined in the beginning of this paper, if we neglect classical effects $\mathcal{O}[2Gm_{1,2}/(c^2 R_{1,2})]$, the space-time metric *before* the measurement $\mathcal{M}_2$, (in the notation of Ref. [4], and replacing r_2 by r) can be written as

$$d\tau^2 \,=\, dt^2 \,-\, dr^2 - r^2\,d\theta \,-\, r^2\,\sin^2\theta\,d\varphi^2 \quad , \tag{5}$$

After the measurement $\mathcal{M}_2$, this space-time metric is changed to

$$d\tau^2 \,=\, \left[1-\frac{1}{r}\left(\frac{2\,G\sqrt{\beta}\,\hbar}{R_2\,c^3}\right)\right]\,dt^2 \,-\, \left[1-\frac{1}{r}\left(\frac{2\,G\sqrt{\beta}\,\hbar}{R_2\,c^3}\right)\right]^{-1}\,dr^2 - r^2\,d\theta \,-\, r^2\,\sin^2\theta\,d\varphi^2 \quad . \tag{6}$$

In reference to the above indicated neglect of classical effects we should note that, as a consequence of the assumption $R_{1,2} \ll \hbar/(m_{1,2}\,c)$, the following inequality holds in the region of interest (i.e., location of particle-1: $r \gtrsim R_2$)

$$\frac{2\,G\,m_2}{c^2\,R_2} \ll \frac{2\,G\sqrt{\beta}\,\hbar}{R_2^2\,c^3} \quad , \tag{7}$$

with a similar relation holding true for the classical influences due to particle-1 on particle-2.

Consequently, as a result of the measurement $\mathcal{M}_2$, the metric of space-time undergoes a change from the form (5) to (6) in an *unavoidable* manner and therefore it matters whether the position measurement on particle-1 is carried before or after the measurement $\mathcal{M}_2$. That is, gravitation and the quantum mechanical character of the measurement process are intertwined in such a manner that the assumption of locality, as specifically expressed in Eq. (1), holds only if one or all of the following are strictly true: $G = 0$, $c = \infty$, and $\hbar = 0$. Admittedly, deviations from locality are exceedingly negligible for measurement processes that involve energies $E \ll m_{pl}c^2$, $m_{pl} \equiv \sqrt{\hbar c/G}$. This is no longer the case for measurement processes where $E \sim m_{pl}c^2$ as may be the case in the early universe and in the vicinity of black holes. The last comment should, however, not be used to argue against our basic conclusion that the measurement process is inherently intertwined with gravitation and locality. While it is true that one does not expect the classical field equations of gravity to be adequate in the description for quantum measurement processes that involve $E \sim m_{pl}c^2$, the in-principle effect survives for much lower energy exchanges (the domain in which gravity may still be treated classically).

In conclusion, therefore, we note that by considering an highly idealized position measurement process, we find that in the strict theoretical sense the fundamental assumption of locality in quantum field theory can only be considered as an approximation. The arguments we present, while directly related to the uncertainty principle and "collapse of wave packet," and hence implicitly connected with the EPR-ideas [5] and the celebrated work of Bell [6], differ from other considerations found in literature [7] in that the role of gravitation in the "hypothesis of localizability" in quantum field theories emerges as a significant element. It should be noted that the essential result on non-commutativity of position measurements, while obtained in an highly stylized situation, seems certain to survive when one or all assumptions of the setup considered are relaxed. Under these relaxed circumstances, of course, other more dominant elements of non-locality may emerge; or it is possible that these other effects may wash out the underlying fundamental gravity-induced non-locality in quantum field theories. This last comment, at present, is pure speculation.

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